1 Aufgaben

a) $f(x) = \sin(x) + \cos(x)$

b) $f(x) = \sin(x) - \cos(x)$

c) $f(x) = \sin(x) \cdot \cos(x)$

d) $f(x) = 3\sin(2x)$

e) $f(x) = (3 + \sin(x))^2$

f) $f(x) = (\sin(x) + \cos(x))^2$

g) $f(x) = \sin^2(x) + \cos^2(x)$

h) $f(x) = \sin^2(x) - \cos^2(x)$

i) $f(x) = \sin^2(x) \cdot \cos^2(x)$

2 Lösungen

a)

$$\begin{aligned} f(x) &= \sin(x) + \cos(x) \\ f'(x) &= \cos(x) - \sin(x) \\ f''(x) &= -\sin(x) - cos(x) \end{aligned}$$

b)

$$\begin{aligned} f(x) &= \sin(x) - \cos(x) \\ f'(x) &= \cos(x) - (-\sin(x)) \\ f'(x) &= \cos(x) + \sin(x) \\ f''(x) &= -\sin(x) + \cos(x) = \cos(x) - \sin(x) \end{aligned}$$

c)

$$\begin{aligned} f(x) &= \sin(x)\cos(x) \\ & \quad \text{Produktregel} \\ f'(x) &= \sin(x) \cdot (\cos(x))' + \cos(x) \cdot (\sin(x))' \\ f'(x) &= -\sin^2(x) + \cos^2(x) \\ & \quad \text{Kettenregel (äußere mal innerer)} \\ f''(x) &= (-2\sin(x) \cdot (\sin(x))') + (2\cos(x) \cdot (\cos(x))') \\ f''(x) &= -2\sin(x)\cos(x) - 2\cos(x)\sin(x) \\ &= -4\sin(x)\cos(x) \end{aligned}$$

d)

$$\begin{aligned} f(x) &= 3\sin(2x) \\ & \quad \text{Kettenregel (äußere mal innerer)} \\ f'(x) &= 3(\cos(2x) \cdot (2x)') \\ f'(x) &= 3(2 \cdot \cos(2x)) \\ f'(x) &= 6\cos(2x) \\ & \quad \text{Kettenregel (äußere mal innerer)} \\ f''(x) &= 6(-\sin(2x) \cdot (2x)') \\ f''(x) &= 6(-2\sin(2x)) \\ f''(x) &= -12\sin(2x) \end{aligned}$$

e)

$$\begin{aligned} f(x) &= (3 + \sin(x))^2 \\ f'(x) &= 2(3 + \sin(x)) \cdot (\sin(x) + 3)' \\ f'(x) &= 2(3 + \sin(x)) \cdot \cos(x) \\ f'(x) &= 6\cos(x) + 2\sin(x)\cos(x) \\ f''(x) &= -6\sin(x) + 2(\sin(x) \cdot (\cos(x))' + (\sin(x))' \cdot \cos(x)) \\ f''(x) &= -6\sin(x) + 2(-\sin^2(x) + \cos^2(x)) \\ f''(x) &= -6\sin(x) + 2\cos^2(x) - 2\sin^2(x) \end{aligned}$$

f)

$$
\begin{aligned}
f(x) &= (\sin(x)+\cos(x))^2 \\
f^{'}(x) &= 2(\sin(x)+\cos(x))\cdot(\sin(x)+\cos(x))' \\
f^{'}(x) &= 2(\sin(x)+\cos(x))\cdot(\cos(x)-\sin(x)) \\
f^{'}(x) &= 2(\cos^2(x)-\sin^2(x)) \\
f^{''}(x) &= 2(2\cos(x)\cdot(\cos(x))'-2\sin(x)\cdot(\sin(x))') \\
f^{''}(x) &= 2(-2\sin(x)\cos(x)-2\sin(x)\cos(x)) \\
f^{''}(x) &= 2(-4\sin(x)\cos(x)) \\
f^{''}(x) &= -8\sin(x)\cos(x)
\end{aligned}
$$

g)

$$
\begin{aligned}
f(x) &= \sin^2(x)+\cos^2(x) \\
f^{'}(x) &= 2\sin(x)\cos(x)+2\cos(x)(-\sin(x)) \\
f^{'}(x) &= 2\sin(x)\cos(x)-2\sin(x)\cos(x) \\
f^{'}(x) &= 0 \\
f^{''}(x) &= 0
\end{aligned}
$$

h)

$$
\begin{aligned}
f(x) &= \sin^2(x)-\cos^2(x) \\
f^{'}(x) &= 2\sin(x)\cos(x)-2\cos(x)(-\sin(x)) \\
f^{'}(x) &= 2\sin(x)\cos(x)+2\sin(x)\cos(x) \\
f^{'}(x) &= 4\sin(x)\cos(x) \\
f^{''}(x) &= 4(\sin(x)(-\sin(x))+\cos(x)\cos(x)) \\
f^{''}(x) &= 4(\cos^2(x)-\sin^2(x))
\end{aligned}
$$

i)

$$
\begin{aligned}
f(x) &= \sin^2(x)\cdot\cos^2(x) \\
f^{'}(x) &= \sin^2(x)(\cos^2(x))+(\sin^2(x))'\cos^2(x) \\
f^{'}(x) &= \sin^2(x)(2\cos(x)\cdot(\cos(x))')+(2\sin(x)\cdot(\sin(x))')\cos^2(x) \\
f^{'}(x) &= \sin^2(x)(2\cos(x)\cdot(-\sin(x)))+(2\sin(x)\cdot\cos(x))\cos^2(x) \\
f^{'}(x) &= -2\sin^3(x)\cos(x)+2\sin(x)\cos^3(x) \\
f^{'}(x) &= 2\sin(x)\cos^3(x)-2\sin^3(x)\cos(x) \\
f^{''}(x) &= 2(\sin(x)\cdot(\cos^3(x))'+(\sin(x))'\cdot\cos^3(x))-2(\sin^3(x)\cdot(\cos(x))'+(\sin^3(x))'\cdot\cos(x)) \\
f^{''}(x) &= 2(\sin(x)\cdot(3\cos^2(x)\cdot(\cos(x))')+\cos(x)\cdot\cos^3(x)) \\
&\quad -2(\sin^3(x)\cdot(-\sin(x))+(3\sin^2(x)\cdot(\sin(x))')\cdot\cos(x)) \\
f^{''}(x) &= 2(\sin(x)\cdot(3\cos^2(x)\cdot(-\sin(x))+cos^4(x))) \\
&\quad -2((-\sin^4(x))+(3\sin^2(x)\cdot\cos(x))\cdot\cos(x)) \\
&= 2(\sin(x)\cdot -3\cos^2(x)\sin(x)+\cos^4(x))-2(-\sin^4(x)+3\sin^2(x)\cos^2(x)) \\
&= 2(-3\cos^2(x)\sin^2(x)+\cos^4(x))-2(3\sin^2(x)\cos^2(x)-\sin^4(x)) \\
&= -6\cos^2(x)\sin^2(x)+2\cos^4(x)-6\sin^2(x)\cos^2(x)+2\sin^4(x) \\
&= 2\sin^4(x)+2\cos^4(x)-12\cos^2(x)\sin^2(x)
\end{aligned}
$$

Quelle: Ableitungen